MAT14: Mathematics for Science and Technology 1A

Why should I do a self-assessed readiness test?

Open Universities Australia subjects in maths

*MAT14: Mathematics for Science and Technology 1A* is based on a fairly standard first level university mathematics curriculum. If you are prepared for it, you’ll experience about the same level of difficulty as any other student at an Australian university who is studying mathematics for use in science, engineering, like disciplines, or simply for its own sake. If you haven’t the background needed, you will very likely find it quite difficult work.

Open Universities Australia offers MAT14 to any student who wishes to do it: that is the philosophy of OUA, and it is endorsed by the MAT14 teaching staff. However, as we are aiming to provide a standard first level university curriculum, we cannot make the work any easier than we have; to reduce its scope would defeat its aim of being able to be broadly recognised as a full first level unit by Australian universities, and lessen its value to you.

The published prerequisite of a background equivalent to Year 12 is necessarily vague; it encompasses many appropriate prior studies. We recognise that it is difficult for many students to interpret. This test will give you a little more certainty about your current mathematical ability and readiness for MAT14.

To help you we are providing this readiness test; one you can do for yourself, so that you can assess whether your existing mathematical skills form an appropriate base for study at the level of MAT14. If you use it, and get our advice that MAT14 might be beyond your current level, you can persist with MAT14 if you wish (knowing you’ve a harder road than some others), you could seek to do preparatory study before tackling MAT14 (advice on this below), or may prefer to take up another field of study. If you get our advice that you appear capable of tackling MAT14, then we believe your skills are sufficient to do it. Though it may still be a deal of work, you shouldn’t stumble for lack of foundation skills.

Other readiness tests available on the internet are at a suitable level: one easily found by a search on its title is ‘are you ready for calculus’. Some tests self-grade, but it is difficult to interpret the grade you get. You may like to refer to another test such as this for a second opinion.

Available preparatory studies

Unilearn UNL31 and UNL32 are mathematics bridging units which will bring you to the level at which you can realistically tackle MAT14. UNL31 is the more basic; it will get you to year 11 level, while UNL32 is recommended to bridge from that level (or a little below) to MAT14’s entry level. Details are available on the Open Universities Australia website.

How do I do the test?

You will need a pen or pencil and blank paper to work on. It is best to work from a printout of this test, doing tasks which you should be able to do if you are ready to study MAT14. Attempt to solve the problems and write an answer to each before referring to the solutions (from page 4). You are asked to *tick boxes* on a score sheet (page 7) as you refer to the solutions, so you’ll need a pen or pencil both to work on the problems and review their solutions. The last section, page 8, gives advice based on the boxes you’ve ticked.
Test problems

Do not look at the solutions until you have attempted all questions. On the score-card (page 7), tick the ☐ if you do not understand a given problem. Once you have attempted all questions without referring to the solutions, you should then look at the solutions for assistance in completing any unfinished problems, and tick the ☐ for that problem. If referring to the solutions still does not help, then tick the ☐ for that problem.

1. The expressions \(\sqrt{x^2}\) and \(x\) are equal for non-negative values of \(x\), and the expressions \(\sqrt[3]{y^2}\) and \(\frac{1}{2} y\) are equal for all \(y\).

Which of the following pairs of expressions are equal for all values of \(x\) and \(y\)? Give a reason, or an example that helps confirm your choice.

(a) \((x^2)^{-1}\) and \(x^2\) or \((x^{-2})^{-1}\) and \(x^{-3}\). Note that the notation \(x^{-2}\) means \(\frac{1}{x^2}\).

(b) \(\ln(e^x)\) and \(x\). Here \(\ln\) is a notation for the natural logarithm, the logarithm of \(x\) to the base \(e \approx 2.71\), sometimes also written as \(\log_e x\).

(c) \(\sqrt{x^2 + y^2}\) and \(x + y\).

(d) \(x^2y \left(\frac{x}{3} + xy^\frac{1}{3}\right)\) and \(x^\frac{3}{2} + x^2y^\frac{3}{4}\).

2. The expression \(x^2 - 3x + 2\) factorises to give \(x^2 - 3x + 2 = (x - 1)(x - 2)\).

Factorise these expressions as far as possible.

(a) \(x^2 - 16\)  
(b) \(x^2 - 8x + 15\)  
(c) \(24x^2 + 12x - 36\)  
(d) \(x^5 - x^3\)

3. The expression \((x - 1)(x^2 + x + 1)\) expands to give that \(x(x^2 + x + 1) + (-1)(x^3 + x + 1) = x^3 + x^2 + x - x^3 - x - 1 = x^3 - 1\).

Expand and simplify each of these expressions.

(a) \((3x - 2y)(5x - 7y)\)  
(b) \((2 + x)^3\)  
(c) \((2x - 5y)^2\)

4. The technique called completing the square of a quadratic gives \(x^2 + 4x - 5 = (x^2 + 4x + 4) - 4 - 5 = (x + 2)^2 - 9\).

So the graph of \(y = x^2 + 4x - 3\) is symmetric about the vertical line \(x = -2\), and has minimum point \((-2, -9)\). As \(x^2 + 4x - 5 = (x - 1)(x + 5)\), its graph has \(x\)-axis intercepts at \(x = 1\) and \(x = -5\).

(a) Sketch the graph \(y = x^2 - 3x + 2\), for \(-2 \leq x \leq 3\). Use factorisation of this quadratic to find its \(x\)-axis intercepts, and complete the square to find its minimum point.

(b) Describe the set of \(x\) values for which \(y > 0\), and indicate this set on the horizontal axis of your graph.

(c) Give the equation of a parabola (there is more than one) with \(x\)-axis intercepts \(-1\) and \(3\).
5. In an algebraic expression, factors common to the numerator and denominator can be cancelled, provided the factor is not zero. So we can simplify \( \frac{x^2 - 4x + 4}{x - 2} \) to \( x - 2 \), provided that \( x \neq 2 \).

(a) Simplify, if possible, \( \frac{2x + 4}{2} \) and \( \frac{2x + 4}{2x} \).
(b) Simplify, if possible, \( \frac{\sqrt{x^2 + x^4}}{x} \) and \( \frac{\sqrt{x + 4}}{x} \).

6. These identities describe properties of a logarithm function (they apply to all logarithm functions) \( \log(ab) = \log a + \log b \), \( \log\left(\frac{a}{b}\right) = \log a - \log b \), \( \log(a^n) = n \log a \).

(a) Use these properties to simplify the expressions \( \log(2) + \log(a) \), \( \log(a^2) \), \( \log(a^{-1}) \) and \( \log\left(\frac{a^2}{b^3}\right) \).
(b) Write \( \log y \) in terms of \( \log x \), if we know that \( y = 3x^2 \).

7. The equation of a straight line with slope \( m \) and \( y \)-axis intercept \( b \) is \( y = mx + b \). So the line \( y = 2x + 3 \) has slope 2 (y changes twice as much as x changes) and passes through the point (0, 3).

(a) On a graph sketch the lines \( y = 2x - 1 \) and \( 4x + 2y - 4 = 0 \).
(b) Give the equation of a line of slope 2 through the point (1, 1).
(c) Give the equation of a line through the points (1, 1) and (-1, 2).

8. The system of equations \( \begin{cases} x + y = 3 \\ 2x + 3y = 5 \end{cases} \) can be solved by writing the first equation as \( y = 3 - x \), and using this in the second equation to get \( 2x + (3 - x) = 5 \), from which we get the systems solution \( x = 4 \) and consequently \( y = -1 \).

(a) \( \begin{cases} 2x + 3y = 11 \\ 2x + 5y = 13 \end{cases} \)
(b) \( \begin{cases} 2x + 5y = 1 \\ 3x + 8y = 1 \end{cases} \)

9. The function \( f(x) = x^3 - 2x^2 + 1 \) has derivative \( f'(x) = \frac{df}{dx} = 3x^2 - 4x \). This gives the slope of the tangent line to the curve with equation \( y = x^3 - 2x^2 \). So the tangent line through the point (0, 1) on this curve has slope \( f'(0) = 3(0)^2 - 4(0) = 0 \), and so is the horizontal line \( y = 1 \). The tangent line to the curve at (1, 0) has slope \(-1\), and so is the line \( y = -x + 1 \).

(a) Find the equation of the tangent line to the curve \( y = x^3 - 2x^2 \) at the point (2, 0).
(b) Write down the derivative of the function \( g(x) = x^3 - 12x \), and locate a point on the curve \( y = x^3 - 12x \) for which this derivative is zero.

10. A quadratic equation \( ax^2 + bx + c = 0 \) is solved by the two interpretations of the formula \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \). So \( x^2 - 3x + 2 = 0 \) has solutions \( x = \frac{3 \pm \sqrt{9 - 4 \cdot 2}}{2} = 2 \) and \( x = \frac{3 - \sqrt{9 - 8}}{2} = 1 \).

(a) Use this method to find both solutions to the equation \( 2x^2 - 5x + 3 = 0 \).
(b) Show that the equation \( 3x^2 - 9x + 7 = 0 \) has no real valued solutions.
Solutions to the problems

As you use these solutions, be sure to mark your scorecard. Put a tick in the square box if you didn’t understand the problem, a tick in the circle if you think you needed the answer given to do the problem; and put a tick in the triangle box if the answer given still doesn’t make sense; we’ll refer to these later on when we give you feedback on your readiness for MAT14.

1. (a) \((x^{-2})^{-1} = \left(\frac{1}{x^2}\right)^{-1} = x^2\) for all \(x\) other than \(x = 0\), so the equality does not hold for all \(x\). When \(x = 0\) the expression \(x^2\) takes the value 0, but the expression \((x^{-2})^{-1}\) is undefined at \(x = 0\) as division by zero is undefined (so \(0^{-2} = \frac{1}{0}\) makes no sense).

And so \((x^{-2})^{-1} = x^2\) is not the same as \(x^{-3}\), unless \(x = 1\).

(b) \(\ln(x^2) = 2\ln(x)\) for all values of \(x\), as the logarithm function undoes the effect of the exponential function (they are inverses). This process of manipulating and simplifying functions is one part of algebra.

(c) \(\sqrt{x^2 + y^2} \neq x + y\), unless one of the numbers \(x\) and \(y\) is zero and the other is non-negative. To see that this is so, observe that for positive values of \(x\) and \(y\), \(\sqrt{x^2 + y^2}\) is the side-length of a right-angled triangle whose sides of lengths \(x\) and \(y\) meet at the right-angle. So \(x + y\) is the sum of two lengths, which for positive \(x\) and \(y\) is greater than the length of the hypotenuse of the triangle.

(d) Using the index property \(x^a x^b = x^{a+b}\) we get that \(x^2 y (x^{\frac{1}{2}} + xy^{\frac{1}{2}}) = xy^2(x^{\frac{1}{2}}) + x^2 y(xy^{\frac{1}{2}}) = x^{\frac{3}{2}}y + x^{\frac{5}{2}} y^2 \neq x^{\frac{3}{2}}y + x^2 y^{\frac{3}{2}}\), except for a few particular values such as \(x = y = 0\) or \(x = y = 1\).

2. This problem has not given any clues or recipes for factorisation, as you are expected to have some of these skills. Sometimes you’ll use pattern matching skills like recognition of a difference of squares or factors related to numerical factors of the polynomial’s constant coefficient (useful in (a) or (b)), algebraic skills (useful in (c) or (d)) or results that link solving \(p(a) = 0\) to a factor \((x - a)\) of the polynomial \(p(x)\) (useful in all).

(a) \(x^2 - 16 = (x - 4)(x + 4)\). [This method is known as a “difference of squares”.]

(b) \(x^2 - 8x + 15 = (x - 3)(x - 5)\) [These factors are found using likely factors of 15.]

(c) \(24x^2 + 12x - 36 = 12(2x^2 + x - 3) = 12(2x + 3)(x - 1)\)

(d) \(x^5 - x^3 = x^3(x^2 - 1) = x^3(x - 1)(x + 1)\)

3. Whether you use all of the steps below, or fewer steps making greater use of mental skill, you ought get the same results. If you know about binomial expansions, they can give a quicker solution again.

(a) \((3x - 2y)(5x - 7y) = 3x(5x - 7y) - 2y(5x - 7y) = 15x^2 - 21xy - 10yx + 14y^2 = 15x^2 - 31xy + 14y^2\).

(b) \((2 + x)^3 = (2+x)(2+x)(2+x) = (2+x)(4+4x+x^2) = 8 + 8x + 2x^2 + 4x + 4x^2 + x^3 = 8 + 12x + 6x^2 + x^3\), or the same can be written directly using a binomial expansion (with binomial coefficients 1\(3\)\(3\)\(1\)) as \(1 \times 2^3 + 3 \times 2^2 x + 3 \times 2x^2 + 1 \times x^3\).

(c) \((2x - 5y)^2 = (2x - 5y)(2x - 5y) = 4x^2 - 20xy + 25y^2\), or as a binomial expansion \(1 \times (2x)^2 + 2 \times (2x)(-5y) + 1 \times (-5y)^2\) which gives the same result.
4. In this problem you need to think graphically; be sure you can answer it without need for a graphics calculator. These are useful sometimes, but you certainly don’t want to be dependant on using one for routine or conceptual graphing skills.

(a) The graph of \( y(x) = x^2 - 3x + 2 = (x - 1)(x - 2) \)
\[= (x - 1.5)^2 - 0.25, \text{ for } -2 \leq x \leq 3 \] is sketched here.

(b) From the graph \( y > 0 \) where \( x < 1 \) or \( x > 2 \); these intervals are depicted on the horizontal axis of the graph. [A separate axis could be used.]

(c) A parabola with \( x \)-axis intercepts \(-1\) and \(3\) has the equation \( y = k(x + 1)(x - 3) = kx^2 - 2kx - 3k \) for any non-zero value of the constant \( k \).

5. (a) We get \( \frac{2x+4}{x} = \frac{2(x+2)}{x} = x + 2 \) and \( \frac{2x+4}{x} = \frac{x+2}{x} \), which can be written as \( 1 + \frac{2}{x} \) (but whether this is considered simpler will depend on what is to be done with it).

(b) We get \( \sqrt{x^2 + 4} = \sqrt{x^2(1+\frac{4}{x^2})} = x\sqrt{1+\frac{4}{x^2}} = \sqrt{1+\frac{4}{x^2}} \) but \( \frac{x+4}{x} \) does not really simplify, though it is equal to expressions like \( \sqrt{\frac{1+\frac{4}{x^2}}{\sqrt{x}}} \) or \( \sqrt{\frac{1}{x} + \frac{4}{x^2}} \).

6. (a) We get \( \log 2 + \log a = \log(2a), \log(a^2) = 2 \log a, \log(a^{-1}) = -\log a \) and \( \log \left( \frac{x^2}{2} \right) = 2 \log a - 4 \log b \).

(b) If we know that \( y = 3x^2 \), then \( \log y = \log(3x^2) = \log 3 + 2 \log x \).

7. (a) The graph here sketches parts of the lines \( y = 2x-1 \) and \( 4x + 2y - 4 = 0 \), which can be written as \( y = -2x + 2 \).

(b) The equation of a line of slope \( 2 \) through the point \( (1,1) \) is of the form \( y = 2x + b \). If \((1,1)\) is on this line, then \( 1 = 2(1) + b \) and so \( b = -1 \). So the line is \( y = 2x - 1 \).

(c) A line through the points \( (1,1) \) and \((-1,2)\) has slope \( m = \frac{-1}{2} = -\frac{1}{2} \), and so has equation \( y - 2 = -\frac{1}{2}(x + 1) \). Alternatively, if the line is \( y = mx + b \) and both points lie on the line, then \( 1 = m(1) + b \) and \( 2 = m(-1) + b \), which can be solved simultaneously to give \( m = -\frac{1}{2} \) and \( b = -\frac{3}{2} \).

8. (a) These two equations differ in that the second equation has extra terms \( 2y \) and \( 2 \); for both equations to hold, we need \( 2y = 2 \) and so \( y = 1 \). The first equation then gives \( x = \frac{1}{2}(11 - 3y) = \frac{1}{2}(11 - 3) = 4 \).

Other valid methods include sketching the graphs of the corresponding lines (as in 7(a)) and finding the point of intersection of the lines, though this will usually have some visual error. Or using the first equation to write \( x = \frac{1}{2}(11 - 3y) \), then using this in the second equation to give \((11 - 3y) + 5y = 13 \), and solving this equation for its variable \( y = 1 \). The first equation again gives \( x = 4 \).
You may also know of other methods of solution, such as Cramer’s rule, Gaussian elimination, or using an inverse matrix.

(b) One solution uses the first equation to write \( x = \frac{1}{2} - \frac{5}{2}y \), and so by the second equation \( 3\left(\frac{1}{2} - \frac{5}{2}y\right) + 8y = 1 \), giving that \((8 - \frac{15}{2})y = 1 - \frac{3}{2}\), and so \( y = -1 \). Thus \( x = \frac{1}{2} + \frac{5}{2} = 3 \).

9. The techniques and theory of differential and integral calculus are built up from scratch in MAT14; but we prefer that you have met these ideas before (likely you will have seen these in less detail). If you’ve never seen a derivative you will experience more difficulty than the students who have already used them. This problem is asking you about derivatives of the easiest functions; polynomials. It should be easy familiar work, and interpreting the values obtained should be simple; if you have never before seen derivatives and this is the only problem on which you are having difficulty, then you should be able to pick up this new concept during your studies for MAT14.

(a) As \( \frac{dy}{dx} = 3x^2 - 4x \), we get \( y'(2) = 4 \), and so the tangent line has slope 4. Its equation has the form \( y = 4x + c \), and since it passes through the point \((2,0)\), we can set the constant \( c = -8 \), giving equation \( y = 4x - 8 \).

(b) Here \( g'(x) = 3x^2 - 12 \). This derivative is zero (giving a horizontal tangent line) when \( x = \pm 2 \); so the point \((2,-16)\) or the point \((-2,16)\) on the graph of \( y = x^3 - 12x \) give a zero derivative.

10. (a) We get \( x = \frac{5 \pm \sqrt{25 - 24}}{4} = \frac{5 \pm 1}{4} \), and so \( x = 1 \) or \( x = \frac{3}{2} \).

(b) We would get \( x = \frac{9 \pm \sqrt{81 - 84}}{6} = \frac{9 \pm \sqrt{-3}}{6} \), but as \( \sqrt{-3} \) is not defined (as a real number) we get no real solutions.
Scorecard

☐ Put a tick in the square box if you do not understand the problem or task sought.

☐ Put a tick in the circle if you think you needed the solution provided to do the problem; that is, you were unable to do it, even though you knew what was needed.

△ Put a tick in the triangle box if the solution provided still doesn’t make sense; that is, if you can’t understand what has been done and how it answers the problem.

1. (a) ☐☐△
   (b) ☐☐△
   (c) ☐☐△
   (d) ☐☐△

2. (a) ☐☐△
   (b) ☐☐△
   (c) ☐☐△
   (d) ☐☐△

3. (a) ☐☐△
   (b) ☐☐△
   (c) ☐☐△

4. (a) ☐☐△
   (b) ☐☐△
   (c) ☐☐△

5. (a) ☐☐△ & ☐☐△
   (b) ☐☐△ & ☐☐△

6. (a) ☐☐△
   (b) ☐☐△

7. (a) ☐☐△
   (b) ☐☐△
   (c) ☐☐△

8. (a) ☐☐△
   (b) ☐☐△

9. (a) ☐☐△
   (b) ☐☐△

10. (a) ☐☐△
     (b) ☐☐△

Now tally your results

Your readiness to do MAT14 will of course be indicated by not very many ticks above. So now do the tallies below; remember that your result is entirely private, the only person who will see these results is you.

- On ....... of the 29 problems, I ticked ☐, because I didn’t understand the task, or couldn’t see how to go about it.

- And on ....... problems, I ticked ☐, because though I was able to get started, and knew what to do, I still was unable to complete the task completely without reference to the solutions.

- And on ....... problems, I ticked △, because I found the solution and explanations given incomprehensible, even when I was reading how I supposedly could do it.
How to interpret your performance

By now you will have realised that we are trying to give you feedback on a number of different aspects of doing and studying mathematics.

- We are testing your familiarity with common mathematical notions, methods and terminology. We are also checking that you have seen certain core ideas previously, where these are assumed skills in MAT14, or where some familiarity with topics is presumed.

- We are trying to test your ability to read and assess what you need to do in mathematical tasks. The solutions are written in a similar style to descriptions of new work you will meet in our printed study materials. There are two issues associated with your ability that you’ll need to consider.
  - We use written language to describe mathematical processes, and use a mixture of English expression and mathematical symbolism in our explanations. The problem solutions are written in the same style as our printed study materials, but refer to skills which we expect to be familiar. The MAT14 printed study materials use this style of writing to convey new topics. It is absolutely essential that you are able to read (easily) these solutions and understand the methods and information they convey. [In MAT14 we also provide some alternatives to working from written descriptions, but only for a few topics.]
  - The solutions here are presented in detail. You will not be provided with the same level of detail on elementary ideas like solving equations or simplifying expressions as we introduce and develop new topics. Whether you’re ready to understand the new topics in MAT14 can be assessed (by you) by determining how easily you are able to work through elementary steps. If you needed very little help from the solutions to the test, and found that they contained just a few points you had not considered before you read them, then you will be able to work on new material without being stopped by small tasks. But if you were unable to anticipate or execute the steps needed in these problems, then you are probably not fully prepared for MAT14, and should consider a ‘bridging unit’.

- There are many mathematical tasks that you have learnt, where you have made use of various forms of technology. If you’ve studied in recent years, you may have developed skill in using and interpreting graphical, or even symbolic, calculator and computer output. You may have used these skills while doing this test. Most of MAT14’s content can be assisted by good use of technology, but also hampered by overdependence on it. Skills in this area can be a great help, but overreliance on this technology can also be a considerable liability.
Now for some advice

If you have ticked a ☐ more than twice, you need to ask yourself whether it was because:

- the language used in the question was unfamiliar, in which case you are likely to be able to pick up the terminology in MAT14 without too much trouble as you study it,

- you have never seen the mathematical concept before. We certainly believe that the mathematical concepts in this diagnostic test should be familiar. In the MAT14 study materials, concepts such as algebraic equality of expressions, factorising, expanding, solving equations (including quadratic equations and pairs of equations solved simultaneously), simplifying, cancellation in algebra, using powers and logarithms and sketching and interpreting graphs, are assumed to be familiar concepts. You will have difficulty if you are not able to apply these techniques without prompting, and you will probably find much of the study material of MAT14 difficult to understand without refreshing your knowledge with a ‘bridging unit’.

If you have ticked a ☐ more than 5 times, then you need to ask yourself whether:

- you had forgotten how to do the particular task, in which case, with some revision and work, you should be able to get up to speed.

- you had never learnt to do that task, in which case it may well be that you have too large a backlog of learning to do to be able to successfully apply effort to the new tasks required in MAT14. In such an instance, we recommend that you take a ‘bridging unit’ before attempting MAT14.

If you have ticked a △ more than once, then you will no doubt also have a great deal of trouble understanding the study material of MAT14. In such a case we recommend that you first undertake some bridging studies.